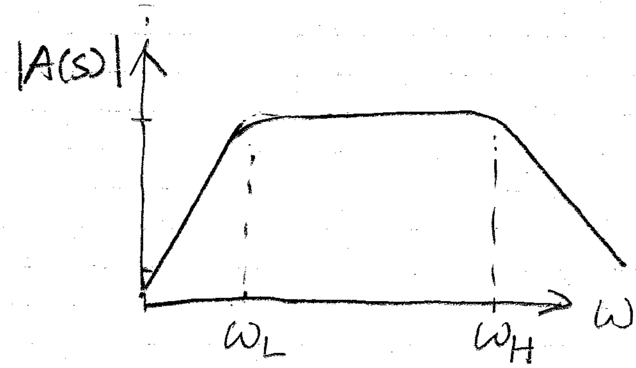
High Frequency Response of CE Amplifiers



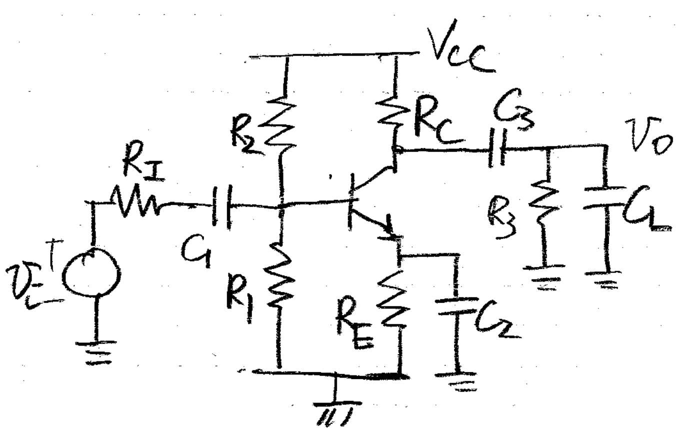
$A(s) = A_{mid}\, F_L(s)\, F_H(s)$

The low frequency response $F_L(s)$ is primarily determined by the bypass and coupling capacitors.

The high frequency response $F_H(s)$ is primarily determined by the intrinsic capacitances of the transistors, such as

$\begin{cases} C_\pi, C_\mu \text{ in BJT} & \cancel{\text{(typically in uF)}} \\ C_{GS}, C_{GD} \text{ in MOS} & \text{(typically < nF)} \end{cases}$

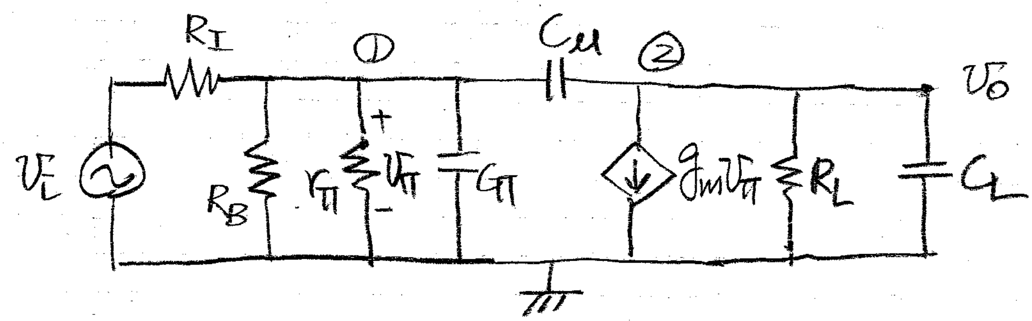
Common-Emitter (CE) amplifier



High-frequency equivalent ckt

① Replace C_1, C_2, C_3 by short ckt

② Use high-frequency hybrid-pi model for BJT



$R_B = R_1 // R_2$

$R_L = r_o // R_C // R_3$

(I) Brute Force solution

Solve the ckt by KVL, KCL

KCL @ ①: $\frac{v_\pi - v_i}{R_I} + \frac{v_\pi}{R_B // r_\pi} + v_\pi s C_\pi + (v_\pi - v_o) s C_\mu = 0$ ③

KCL @ ②: $(v_o - v_\pi) s C_\mu + g_m v_\pi + \frac{v_o}{R_L} + s C_L v_o = 0$ ④

From ④ $v_o (s C_\mu + s C_L + \frac{1}{R_L}) = - v_\pi (g_m - s C_\mu)$

From ③ $v_\pi (\frac{1}{R_I // R_B // r_\pi} + s C_\pi + s C_\mu) = v_o s C_\mu + \frac{v_i}{R_I}$

Let $R_I // R_B // r_\pi = R'$

$$\Rightarrow v_o \left(\frac{1}{R_L} + s(C_\mu + C_L)\right) = \frac{-(g_m - C_\mu)(s C_\mu \cdot v_o + \frac{v_i}{R_I})}{\frac{1}{R'} + s(C_\pi + C_\mu)}$$

$$\Rightarrow A(s) = \frac{v_o}{v_i} = -g_m R_L \frac{R'}{R_I} \cdot \frac{(1 - s \frac{C_\mu}{g_m})}{f(s)}$$

$$f(s) = 1 + s[R'(C_\pi + C_\mu + g_m R_L C_\mu) + R_L (C_\mu + C_L)] + s^2 [R_L R' (C_\mu C_\pi + C_L C_\pi + C_L C_\mu)]$$

$$F_H(s) = \frac{1 + \frac{s}{\omega_z}}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})}$$

RHP zero : $\omega_Z = \frac{g_m}{C_\mu} > \omega_T = \frac{g_m}{C_\mu + C_\pi}$

much higher than ω_T ($\because C_\pi \gg C_\mu$)
$\Rightarrow$ ignore

Assume there is a dominant pole.

$$D(s) = \left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)$$
$$= 1 + s\left(\frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}}\right) + \frac{s^2}{\omega_{p1}\omega_{p2}}$$

$\omega_{p2} \gg \omega_{p1}$

$\frac{1}{\omega_{p2}} \ll \frac{1}{\omega_{p1}}$

$$D(s) \approx 1 + \frac{s}{\omega_{p1}} + \frac{s^2}{\omega_{p1}\omega_{p2}} = 1 + A_1 s + A_2 s^2$$

$$\Rightarrow \omega_{p1} = \frac{1}{A_1} = \left[R'(C_\pi + (1+g_m R_L) C_\mu) + R_L (C_\mu + C_L)\right]^{-1}$$

$$\omega_{p2} = \frac{1}{A_2/A_1} = \frac{A_1}{A_2} = \frac{R'(C_\pi + (1+g_m R_L) C_\mu + R_L (C_\mu + C_L)}{R_L R' (C_\mu C_\pi + C_L C_\pi + C_L C_\mu)}$$

Need to verify $\omega_{p2} \gg \omega_{p1}$

If we assume $C_\pi \gg C_\mu, C_L$

$$\omega_{p2} \gtrsim \frac{\cancel{R'}(1+g_m R_L)\cancel{C_\mu}}{R_L \cancel{R'} C_\pi \cancel{C_\mu}} \approx \frac{g_m}{C_\pi} > \omega_T = \frac{g_m}{C_\mu + C_\pi}$$

The dominant pole ω_{p1}

$$\frac{1}{\omega_{p1}} = R' C_\pi + R'(1+g_m R_L) C_\mu + R_L (C_\mu + C_L)$$

: Sum of RC times.

Open-Circuit Time Constant (OCTC) Method

Approximation to quickly find dominant pole in high frequency response

$$F_H(s) = \frac{(1+\frac{s}{\omega_{z1}})(1+\frac{s}{\omega_{z2}})(\cdots)}{(1+\frac{s}{\omega_{p1}})(1+\frac{s}{\omega_{p2}})(\cdots)} = \frac{1+a_1 s+a_2 s^2+\cdots}{1+b_1 s+b_2 s^2+\cdots}$$

$$b_1 = \frac{1}{\omega_{p1}} + \frac{1}{\omega_{p2}} + \cdots\cdots = \sum_{1}^{n_p} \frac{1}{\omega_{pi}} = \sum_{i=1}^{n_p} \tau_{pi}$$

Through network theory, one can prove

$$\sum_{i=1}^{n_p} \tau_{pi} = \sum_{j} R_{jo} C_j$$

C_j = capacitors in H.F. circuit

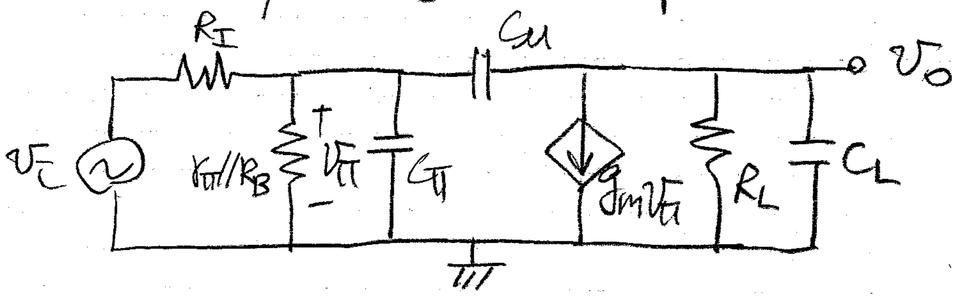
R_{jo}: resistances measured at terminals of C_j, with other capacitors open-circuited.
and all independent voltage sources short-circuited,
all independent current sources open-circuited.

Now use dominant pole approximation:

$$b_1 \approx \frac{1}{\omega_H}$$

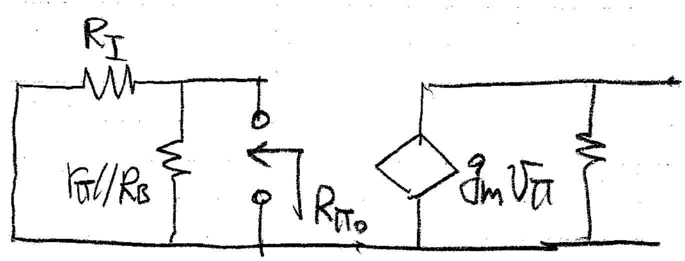
$$\omega_H = \frac{1}{b_1} = \frac{1}{\sum_j R_{jo} C_j}$$

OCTC analysis of CE amplifier



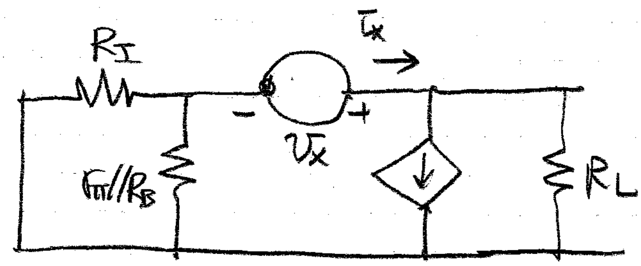
$$\omega_H = \frac{1}{R_{\pi o} C_\pi + R_{\mu o} C_\mu + R_{Lo} C_L}$$

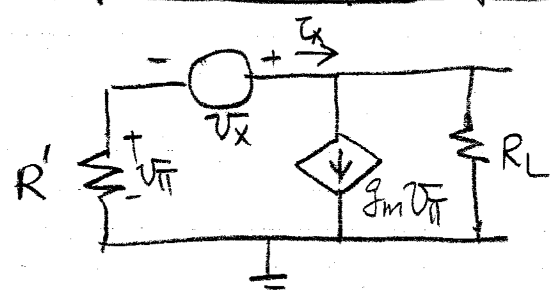
$R_{\pi o}$:



$$R_{\pi o} = r_\pi // R_B // R_I = R'$$

$R_{\mu o}$:





$$v_\pi = -i_x R'$$

KCL: $i_x = g_m v_\pi + (v_\pi + v_x)/R_L$

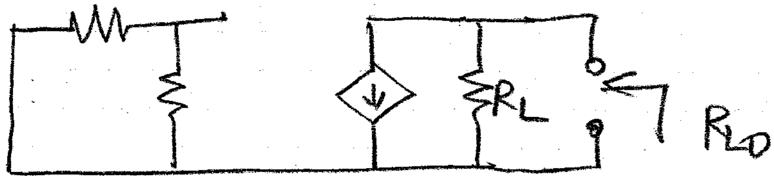
$$i_x = \frac{1}{R_L}(1 + g_m R_L)\, v_\pi + \frac{v_x}{R_L}$$

$$[R_L + R'(1 + g_m R_L)]\, i_x = v_x$$

$$R_{\mu o} = R_L + R'(1 + g_m R_L)$$

TOPS

R_{LO}:



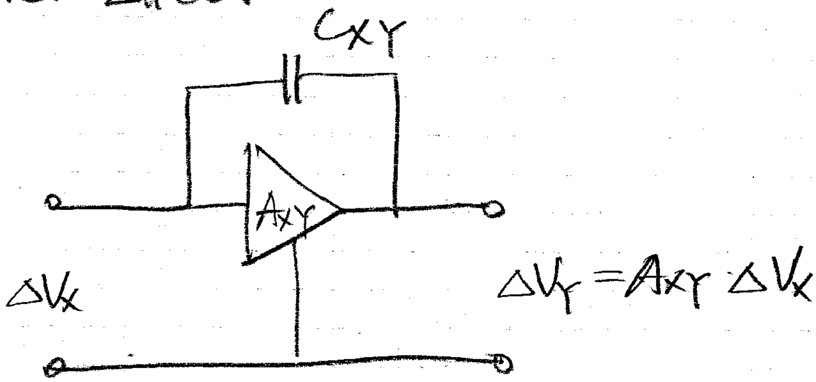
$v_\pi = 0 \quad g_m v_\pi = 0$

$R_{LO} = R_L$

$$\Rightarrow \omega_H = \frac{1}{R' C_\pi + (R_L + R'(1 + g_m R_L)) C_\mu + R_L C_L}$$

Same as brute force solution!
But much easier to analyze.

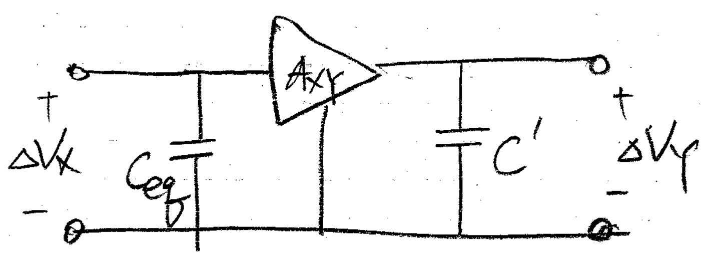
Miller Effect



$$C = \frac{\Delta Q}{\Delta V}$$

$$\Delta Q = C_{XY}(\Delta V_X - \Delta V_Y) = C_{XY}(1 - A_{XY})\Delta V_X$$

Can be considered as an equivalent capacitance on the input side:



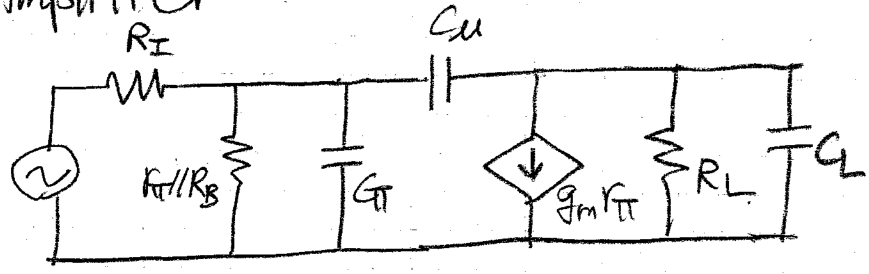
$C_{eq} = (1 - A_{XY})C_{XY}$ ⇒ called "Miller Capacitance"

On the output side

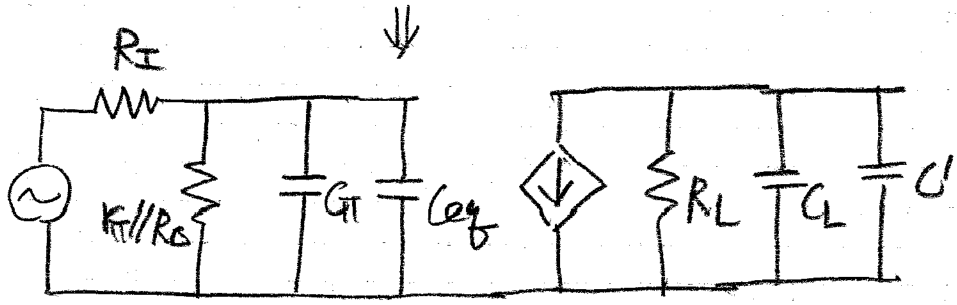
$$\Delta Q = C_{XY}(\Delta V_Y - \Delta V_X) = C_{XY}\left(1 - \frac{1}{A_{XY}}\right)\Delta V_Y$$

$$\Rightarrow C' = \left(1 - \frac{1}{A_{XY}}\right)C_{XY}$$

CE Amplifier



$\Downarrow$



$$C_{eq} = (1 - A_{xy}) C_\mu = (1 - (-g_m R_L)) C_\mu$$
$$= (1 + g_m R_L) C_\mu$$
$$C' = \left(1 - \frac{1}{A_{xy}}\right) C_\mu = \left(1 + \frac{1}{g_m R_L}\right) C_\mu \approx C_\mu$$

$$A(s) = \frac{r_\pi // R_B}{R_I + r_\pi // R_B} \cdot (-g_m R_L) \cdot \frac{1}{[1 + sR'(C_\pi + C_{eq})][1 + sR_L(C_L + C')]}$$

$$R' = R_I // r_\pi // R_B$$

Dominant pole

$$\omega_H = \frac{1}{R'(C_\pi + C_{eq}) + R_L(C_L + C')}$$
$$= \frac{1}{R'(C_\pi + (1 + g_m R_L) C_\mu) + R_L(C_L + C_\mu)}$$

Same as brute force OCTC